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A COMPUTER PROGRAM TO IMPLEMENT THE CHEN METHOD OF DIMENSIONAL ANALYSIS

BY

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JANUARY 1990

FINAL REPORT
MAY 1989 - JUNE 1989

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INDIVIDUAL PROTECTION DIRECTORATE

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REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; Distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) NATICK/TR-90/017	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION U.S. Army Natick RD&E Center	6b. OFFICE SYMBOL (If applicable) STRNC-ITFF
7a. NAME OF MONITORING ORGANIZATION	
6c. ADDRESS (City, State, and ZIP Code) Kansas Street Natick, MA 01760-5019	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)
9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
1L162723	AH98	CC	015

11. TITLE (Include Security Classification)
A Computer Program to Implement the Chen Method of Dimensional Analysis

12. PERSONAL AUTHOR(S)
Philip Cunniff

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final	FROM May 89 TO Jun 89	1990 January	[illegible]

16. SUPPLEMENTARY NOTATION

17. COSATI CODES		
FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
COMPUTER PROGRAMMING, CHEN METHOD, DATA, MATRIX ALGEBRA, DIMENSIONAL ANALYSIS, FORTRAN 77, PROGRAMMING LANGUAGES

19. ABSTRACT (Continue on reverse if necessary and identify by block number)
A computer program to implement the Chen method of dimensional analysis is presented. The Chen method allows for relatively easy determination of dimensional groups using matrix algebra. The computer program to implement the method provides dimensional groups in simple format and is written in standard Fortran 77. Several examples of the use of the program are provided.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Philip Cunniff	22b. TELEPHONE (Include Area Code) 508-651-5463 — 22c. OFFICE SYMBOL STRNC-ITFF

DD Form 1473, JUN 86 — Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

Preface

The computer program undertaken in this project was developed during the period May 1989 to June 1989. The work was funded under Program element No. 1L162723, Project No. AH98, Task No. CC, Work Unit Accession No. 15.

Table of Contents

A COMPUTER PROGRAM TO IMPLEMENT THE CHEN METHOD OF DIMENSIONAL ANALYSIS

Introduction

Dimensional analysis provides a method of reducing the experimentation required to analyze a problem of several variables by illustrating the nature of the relationship between the variables. Various methods exist to determine dimensionless parameters, but the methods all have in common the following requirements:

1. All pertinent variables of the problem must be included in the analysis,
2. No extraneous variables may be included in the analysis,
3. Only a single relationship exists between the variables of the problem.

The procedures for determining dimensional parameters do not require knowledge of the relationships between the variables of the problem but the results of the procedures are often improved if some knowledge is applied.

Assuming that sufficient information about the problem is available to satisfy the requirements of the analysis, a relationship between the variables may be written in terms of some unknown function (Φ) as:

$$\Phi(q_1, q_2, q_3, \ldots, q_n) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where the q_i's are the variables of the problem.

Dimensional analysis provides a method of reducing the problem of Equation 1 to the form:

$$\Phi(\pi_1, \pi_2, \pi_3, \ldots, \pi_m) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where the π_i's are dimensionless parameters, and m is less than n.

Various methods for determining dimensionless parameters are available in the literature, most notably the Buckingham Pi method and the Lord Rayleigh method. These methods may be somewhat cumbersome when the number of variables becomes large. Chen [1] recently published a method for determining dimensionless parameters, which uses matrix algebra and is amenable to computer solution.

Chen Method

The Chen method allows for the development of dimensionless groups using standard matrix algebra. The method is illustrated here using an example provided by Chen [2]. Chen considered the problem of forced flow inside a rough circular pipe. The problem statement is given in terms of an unknown function Φ of the important variables of the problem by:

$$\Phi(\Delta p/L,\ D,\ \mu,\ d,\ v,\ L,\ k) = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

where:

$\Delta p/L$ - Pressure drop per unit length of pipe. $[\Delta p/L] = M^1L^{-2}T^{-2}$

D - Pipe Diameter. $[D] = M^0L^1T^0$

μ - Fluid dynamic viscosity. $[\mu] = M^1L^{-1}T^{-1}$

d - Fluid density. $[d] = M^1L^{-3}T^0$

v - Fluid mean velocity. $[v] = M^0L^1T^{-1}$

L - Distance from entrance. $[L] = M^0L^1T^0$

k - Pipe roughness. $[k] = M^0L^1T^0$

In the notation above expressions of the form $[a] = M^1L^1T^{-2}$ should read "a has units of mass and length per unit time squared." Chen showed the dimensional analysis problem in this case may be cast in the following form:

$$\begin{bmatrix} 1 & -2 & -2 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} M \\ L \\ T \end{bmatrix} = \begin{bmatrix} \Delta p/L \\ D \\ v \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} M \\ L \\ T \end{bmatrix} = \begin{bmatrix} d \\ \mu \\ L \\ k \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

The terms in the column matrix are strictly logarithmic terms, and Equations 4 and 5 are a statement of the dimensions of the variables of the problem. Chen termed the matrix of Equation 4 the repeating variable matrix, since the column matrix of Equation 4 contains the repeating terms of the Buckingham Pi method. Note that Equation 4 has the form $\mathbf{Ax} = \mathbf{b}$, so that $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$, where $\mathbf{A}^{-1}$ is the inverse of the matrix $\mathbf{A}$. Equation 5 has the form $\mathbf{Bx} = \mathbf{c}$, and consequently $\mathbf{BA}^{-1}\mathbf{b} = \mathbf{c}$, so that:

$$\begin{bmatrix} 1 & -3 & 0 \\ 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{A}^{-1}\mathbf{b} = \begin{bmatrix} d \\ \mu \\ L \\ k \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

where $\mathbf{A}^{-1}$ is the inverse of the matrix of Equation 4 and $\mathbf{b}$ is the column vector of Equation 4. Expanding Equation 6 gives:

$$\begin{bmatrix} 1 & -3 & 0 \\ 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & -4 & -2 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \Delta p/L \\ D \\ v \end{bmatrix} = \begin{bmatrix} d \\ \mu \\ L \\ k \end{bmatrix} \quad \ldots \quad (7)$$

Carrying out the matrix multiplication gives:

$$\begin{bmatrix} 1 & 1 & -2 \\ 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta p/L \\ D \\ v \end{bmatrix} = \begin{bmatrix} d \\ \mu \\ L \\ k \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

Equation 8 provides four dimensionless groups as follows:

$$\pi_1 = \frac{dv^2L}{\Delta pD} \qquad \pi_2 = \frac{\mu vL}{\Delta pD^2} \qquad \pi_3 = L/D \qquad \pi_4 = k/D$$

The Chen method has been programmed in the Fortran computer programming language for execution under the Unix operating system. Examples of the use of the computer program are provided in the following section. A listing of the computer program is provided in the Appendix.

Examples of Computer Program Use

The computer program was developed to operate under the Unix operating system. The program reads from the standard input and writes to the standard output. A data file of the input data is usually created, and input redirection used to execute the program. The format of the input file is discussed in the examples.

EXAMPLE 1 -- Drag Force on Sphere

The variables of interest in this problem are:

Quantity	Symbol	Dimensions			
		F	M	L	T
Drag Force	D	1	0	0	0
Diameter of Sphere	d	0	0	1	0
Density of Air	p	0	1	-3	0
Viscosity of Air	μ	1	0	-2	1
Velocity of Air	v	0	0	1	-1
Newton's Proportionality Constant	g_c	-1	1	1	-2

The table above is a statement of the dimensions of the variables of the problem. To carry out the dimensional analysis, it is necessary to determine which of these variables will be the repeating variables. The selection of repeating variables is not completely arbitrary; the repeating variable matrix must not be singular or ill-conditioned. The computer program tests for these conditions, however, and improper selection of repeating variables will result in an error message from the computer program.

Input Data

The general format of the input file is:

Number of variables of the problem.
Number of variables in the repeating variable matrix
Primary quantity number 1
Primary quantity number 2

.
.
.

Primary quantity number n
a b c d ... n Variable name i
a b c d ... n Variable name j
a b c d ... n Variable name k

.
.
.

a b c d ... n Variable name m

where a, b, c, etc. are the dimensions of primary quantities in variable i, j, etc. In the example above, the primary quantities are force, mass, length, and time; the variable g_c has primary quantities $MLT^{-2}F^{-1}$.

For this problem, the input file is:

```
6
4
force
mass
length
time
-1.    1.     1.    -2.   gc
 0.    0.     1.    -1.   v
 0.    1.    -3.     0.   p
 0.    0.     1.     0.   d
 1.    0.    -2.     1.   μ
 1.    0.     0.     0.   D
```

Note that the order of the input file assumes that the repeating variables for this problem are g_c, v, p, and d. Alternate formulations of the problem may not produce the same results.

Program Output

```
number of variables for this problem:   6
number of variables in the repeating variable matrix:   4

 REPEATING VARIABLE MATRIX
        force       mass        length      time

        -1.0         1.0         1.0        -2.0        gc
          .0          .0         1.0        -1.0        v
          .0         1.0        -3.0          .0        p
          .0          .0         1.0          .0        d


 INDEPENDENT VARIABLE MATRIX
        force       mass        length      time

         1.0          .0        -2.0         1.0        μ
         1.0          .0          .0          .0        D

INVERSE OF THE REPEATING VARIABLE MATRIX
        -1.0000        2.0000        1.0000        2.0000
          .0000         .0000        1.0000        3.0000
          .0000         .0000         .0000        1.0000
          .0000       -1.0000         .0000        1.0000

SOLUTION OF THE PROBLEM
        gc            v            p            d

      -1.000        1.000        1.000        1.000            μ
      -1.000        2.000        1.000        2.000            D
```

The drag force on a sphere in a uniform flow field may expressed as:

$$\Phi\left(\frac{vpd}{g_c\mu}\ ,\ \frac{Dg_c}{v^2pd^2}\right) = 0$$

Where the first dimensionless parameter is the Reynolds Number, and the second dimensionless parameter is the coefficient of drag for the sphere. The problem is now understood to be: the coefficient of drag on a sphere in a flow field is a function of Reynolds number only.

EXAMPLE 2 -- Centrifugal Pump Performance

Schnittger [3] considered the problem of centrifugal pump performance; the variables of interest are:

Quantity	Symbol	Dimensions M	L	T
Pump Flow Rate	Q	0	3	-1
Pressure Rise in Pump	Δp	1	-1	-2
Pump Impeller Diameter	D	0	1	0
Pump Speed	N	0	1	-1
Power Input to Pump	E	1	2	-3
Density of Fluid	rho	1	-3	0
FLUID Dynamic Viscosity	μ	1	-1	-1

Input data

```
7
3
mass
length
time
 0.   0.   -1.   N
 0.   1.    0.   D
 1.  -3.    0.   rho
 1.  -1.   -1.   μ
 1.   2.   -3.   E
 0.   3.   -1.   Q
 1.  -1.   -2.   Δp
```

Program Output

```
number of variables for this problem:    7
number of variables in the repeating variable matrix:    3

 REPEATING VARIABLE MATRIX
        mass         length       time

          .0            .0         -1.0         N
          .0           1.0           .0         D
         1.0          -3.0           .0         rho


 INDEPENDENT VARIABLE MATRIX
        mass         length       time

         1.0          -1.0         -1.0         μ
         1.0           2.0         -3.0         E
          .0           3.0         -1.0         Q
         1.0          -1.0         -2.0         Δp

INVERSE OF THE REPEATING VARIABLE MATRIX
           .0000          3.0000          1.0000
           .0000          1.0000           .0000
        -1.0000            .0000           .0000

SOLUTION OF THE PROBLEM
          N             D            rho

       1.000         2.000         1.000              μ
       3.000         5.000         1.000              E
       1.000         3.000          .000              Q
       2.000         2.000         1.000              Δp
```

Centrifugal pumps may be compared based on four dimensionless parameters, namely:

Flow Number: $C_Q = Q/(N\,D^3)$

Power Number: $C_E = E/(\text{rho}\,N^3\,D^5)$

Pressure Number: $C_P = \Delta p/(\text{rho}\,N^2\,D^2)$

Reynolds Number: $R_e = \text{rho}\,D^2\,N/\mu$

An alternate formulation of the same problem is presented below. In this case the variables D, rho, μ and N are taken to be the variables of the repeating variable matrix.

Input Data

```
7
3
mass
length
time
 0.   1.    0.    D
 1.  -3.    0.    rho
 1.  -1.   -1.    μ
 0.   0.   -1.    N
 1.   2.   -3.    E
 0.   3.   -1.    Q
 1.  -1.   -2.    Δp
```

Program Output

```
number of variables for this problem:   7
number of variables in the repeating variable matrix:   3

 REPEATING VARIABLE MATRIX
        mass          length        time

          .0           1.0            .0        D
         1.0          -3.0            .0        rho
         1.0          -1.0          -1.0        μ


 INDEPENDENT VARIABLE MATRIX
        mass          length        time

          .0            .0          -1.0        N
         1.0           2.0          -3.0        E
          .0           3.0          -1.0        Q
         1.0          -1.0          -2.0        Δp

INVERSE OF THE REPEATING VARIABLE MATRIX
          3.0000          1.0000            .0000
          1.0000           .0000            .0000
          2.0000          1.0000          -1.0000

SOLUTION OF THE PROBLEM
          D             rho          μ

      -2.000        -1.000        1.000             N
      -1.000        -2.000        3.000             E
       1.000        -1.000        1.000             Q
      -2.000        -1.000        2.000             Δp
```

Here the dimensionless groups are:

π_1 = N D^2 rho/μ
π_2 = E D rho^2/μ^3
π_3 = Q rho/D μ
π_4 = Δp D^2 rho/μ

Example 3 -- Vibration of a Liquid Drop [2]

Taylor [4] presented the following problem where he sought to find the lowest natural frequency Ω of a vibrating liquid drop. The variables of interest for this problem are:

Quantity	Symbol	Dimensions F	M	L	T
Natural Frequency	Ω	0	0	0	-1
Diameter of Drop	d	0	0	1	0
Density of Liquid	p	0	1	-3	0
Surface Tension	α	1	0	-1	0
Newton's Proportionality Constant	g_c	-1	1	1	-2

Having assembled the table above, it is now necessary only to execute the computer program:

Input Data

```
5
4
force
mass
length
time
 0.    0.    1.    0.   d
 0.    1.   -3.    0.   p
-1.    1.    1.   -2.   gc
 1.    0.   -1.    0.   α
 0.    0.    0.   -1.   Ω
```

Program Output

```
number of variables for this problem:   5
number of variables in the repeating variable matrix:   4
```

REPEATING VARIABLE MATRIX

	force	mass	length	time	
	.0	.0	1.0	.0	d
	.0	1.0	-3.0	.0	p
	-1.0	1.0	1.0	-2.0	g_c
	1.0	.0	-1.0	.0	α

INDEPENDENT VARIABLE MATRIX

force	mass	length	time	
.0	.0	.0	-1.0	Ω

INVERSE OF THE REPEATING VARIABLE MATRIX

1.0000	.0000	.0000	1.0000
3.0000	1.0000	.0000	.0000
1.0000	.0000	.0000	.0000
1.5000	.5000	-.5000	-.5000

SOLUTION OF THE PROBLEM

d	p	g_c	α	
-1.500	-.500	.500	.500	Ω

The natural frequency varies inversely as the three-halves power of the drop size.

The dimensionless group may be written as:

$$\Phi\left(\frac{\Omega^2 d^3 p}{g_c \alpha}\right) = 0$$

Summary

The Chen method of dimensional analysis has been shown to be a substantial improvement over other methods of dimensional analysis both in terms of the speed of implementation of the method and in terms of the elegance of the method. A computer program to implement the method has been presented, which should allow for better design of experiments.

References

1. Chen J.J.J., Notes on Dimensional Analysis. Int. J. Mech. Engr. Educ., 16:2 (1989)

2. Chen J.J.J., An Improvement on the Quraishi-Fahidy Method of Dimensional Analysis. Can. J. Chem. Engr., 66 (1988)

3. Schnittger J.R., Dimensional Analysis in Design. J. Vib. Acoust. Stress, and Reliab. Des. 110 (1988)

4. Taylor E.S., Dimensional Analysis for Engineers. Clarendon Press (1974)

APPENDIX

Computer Program Listing

```
      character*20 name(20),dimension(dummy)
      dimension A(dummy,dummy),X(dummy),j2(dummy),i2(dummy),B(20,20),c(20,20)
      read(*,*)M
      read(*,*)N
      write(*,3)M
      write(*,4)N
3      format('number of variables for this problem: ',I3)
4      format('number of variables in the repeating variable matrix: ',I3)
      MminusN = M-N
      do 75 i=1,N
75      read(*,5)dimension(i)
5      format(a20)
      do 100 i=1,N
      do 100 j=1,N
        read(*,10)(A(i,j), j=1,dummy),name(i)
10      format(dummyf5.1,a20)
100      continue
      write(*,*)
      write(*,*)'REPEATING VARIABLE MATRIX'
      write(*,6)(dimension(i), i=1,N)
      do 105 i=1,N
        write(*,11)(A(i,j), j=1,N),name(i)
11        format(dummyf12.1,5x,a20)
105      continue
      write(*,*)
      do 110 i=1,MminusN
      do 110 j=1,N
      k = i+N
        read(*,10)(B(i,j),j=1,dummy),name(k)
110      continue
      write(*,*)
      write(*,*)'INDEPENDENT VARIABLE MATRIX'
      write(*,6)(dimension(i), i=1,N)
6      format(/,2x,dummy(6x,a6),/)
      do 106 i=1,MminusN
        k = i+N

        write(*,11)(B(i,j), j=1,N),name(k)
106      continue
      call minv(A,dummy,E,K,X,J2,I2)
      if (k .ne. 0) then
      write(*,*)'MATRIX ILL CONDITIONED OR SINGULAR, k = ', k
      write(*,*)'E = ',E
      stop
      endif
      write(*,15)
C
```

```
15          format(/,'INVERSE OF THE REPEATING VARIABLE MATRIX',/)
          do 200 i=1,dummy
200          write(*,20) (A(i,j), j=1,dummy)
20          format(dummyf15.4)
          do 300 i=1,MminusN
          do 300 j=1,N
          c(i,j) = 0.0
          do 250 k=1,N
250          c(i,j) = c(i,j) + B(i,k)*A(k,j)
300          continue
          write(*,25)
25          format(/,'SOLUTION OF THE PROBLEM',/)
          write(*,26)(name(i), i=1,N)
26          format(4x,dummy(6x,a6),/)
          do 400 i=1,MminusN
          k = i+N
400          write(*,27) (c(i,j), j=1,dummy),name(k)
27          format(dummyf12.3,10x,a6)
          end

      SUBROUTINE MINV  (A,N,E,K,X,J2,I2)
C*     MATRIX INVERSION ROUTINE-FORMULATED BY E. G. CLAYTON
C
C      A--SQUARE ARRAY (SINGLE PRECISION) CONTAINING ORIGINAL MATRIX
C      N--ORDER OF ORIGINAL MATRIX
C      E--TEST CRITERION FOR NEAR ZERO DIVISOR
C      K--LOCATION FOR SINGULARITY OR ILL-CONDITION INDICATOR
C         K=0 =) MATRIX NONSINGULAR.
C         K=1 =) MATRIX SINGULAR (OR ILL-CONDITIONED)
C      X--A WORK VECTOR OF SIZE N
C      J2--AN INTEGER WORK VECTOR OF SIZE N
C      I2--AN INTEGER WORK VECTOR OF SIZE N
C
      DIMENSION A(N,N),X(N),J2(N),I2(N)
C
C     INITIALIZATION
C
      K=1
      I2(1)=0
      J2(1)=0
C
C     BEGIN COMPUTATION OF INVERSE
C
      DO 15 L=1,N
C
```

```
      L1=L-1
      IF(L1.EQ.0)L1=1
      BIGA=-1.0
C
C     LOOK FOR THE ELEMENT OF GREATEST ABSOLUTE VALUE,CHOOSING
C     ONE FROM A ROW AND COLUMN NOT PREVIOUSLY USED.
C
      DO 5 I=1,N
      DO 1 I3=1,L1
      IF(I .EQ. I2(I3))  GO TO 5
    1 CONTINUE
      DO 4 J=1,N
      DO 2 I3=1,L1
      IF(J .EQ. J2(I3))  GO TO 4
    2 CONTINUE
      AT=ABS(A(I,J))
      IF(BIGA .GE. AT)  GO TO 4
      BIGA=AT
      J1=J
      I1=I
    4 CONTINUE
    5 CONTINUE
C
C     TAG THE ROW AND COLUMN FROM WHICH THE ELEMENT IS CHOSEN.
C
      J2(L)=J1
      I2(L)=I1
      DIV=A(I1,J1)
C
C     TEST ELEMENT AGAINST ZERO CRITERION
      IF(ABS(DIV) .LE. E)  GO TO 221
C
C     PERFORM THE COMPUTATIONS
C
      OOD = 1./DIV
      DO 7 J=1,N
      A(I1,J)=A(I1,J)*OOD
    7 CONTINUE
      A(I1,J1)=OOD
      DO 11 I=1,N
      IF(I1 .EQ. I)  GO TO 11
      DO 10 J=1,N
      IF(J1 .EQ. J)  GO TO 10
      A(I,J)=A(I,J)-A(I1,J)*A(I,J1)
   10 CONTINUE
   11 CONTINUE
C
```

```
      DO 14 I=1,N
      IF(I1 .EQ. I)  GO TO 14
      A(I,J1)=-A(I,J1)*A(I1,J1)
   14 CONTINUE
   15 CONTINUE
C
C     COMPUTATION COMPLETE AT THIS POINT
C     UNSCRAMBLE THE INVERSE
C
      DO 18 J=1,N
      DO 16 I=1,N
      I1=I2(I)
      J1=J2(I)
      X(J1)=A(I1,J)
   16 CONTINUE
      DO 17 I=1,N
      A(I,J)=X(I)
   17 CONTINUE
   18 CONTINUE
      DO 21 I=1,N
      DO 19 J=1,N
      I1=I2(J)
      J1=J2(J)
      X(I1)=A(I,J1)
   19 CONTINUE
      DO 20 J=1,N
      A(I,J)=X(J)
   20 CONTINUE
   21 CONTINUE
      K=0
  221 RETURN
      END
```

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